Force as a Function of Time only \([f(t)]\)

1. [Davis 2.3] A body with mass \(m\) is initially at rest at the origin \((v_0 = 0, x_0 = 0)\). At time \(t = 0\), a force is applied that increases quadratically with time, given by

\[
F = ct^2
\]

Solve for the position of the body, \(x\), as a function of time.

2. [Davis 2.7] A body of mass \(m\) is initially at rest at the origin. Beginning at time, \(t = 0\), a constant force \(F_1\) acts on the body for a time \(t_1\). The force then increases linearly with time so that at time \(t_2 = 2t_1\), the force is \(2F_1\).
   (a) Find an explicit expression for the force during the time \(t_1 < t < t_2\).
   (b) Find the position and velocity of the body at time \(t_2\).

3. A sphere is electrostatically charged and placed in a region of constant electric field \(E_0\) \((F = QE_0)\). The electrostatic charge leaks off slowly, so that the charge on the sphere can be expressed by the relation

\[
Q = Q_0e^{-t/\tau}
\]

Ignoring gravity, determine the velocity and position of the charged sphere as a function of time, assuming that the sphere is at rest at \(x = 0\) at time \(t = 0\). Determine the form of the equations of motion [both \(x(t)\) and \(v(t)\)] in the limit as \(t \to \infty\).

4. [Davis 2.9] A body of mass \(m\) is initially at rest. Starting at time \(t = 0\), it is subject to a force given by

\[
F = F_0e^{-\alpha t}\cos(\omega t + \phi)
\]

Solve for the position, \(x\), and the velocity, \(v\), as a function of time and notice the effect of the initial phase angle \(\phi\).

[Hint: Solve this problem by writing the force as

\[
F = F_0e^{-\alpha t} \Re(e^{i(\omega t + \phi)}) = F_0e^{i\phi}e^{(i\omega - \alpha)t} = F_0e^{i\phi}e^{\beta t}
\]

where \(\Re[f]\) stands for the “real part” of the function \(f\). To obtain the correct final answer, you simply take the “real part” of the answer you obtain using this “trick”. This is most easily accomplished by writing any complex quantity in terms of the magnitude and phase of that complex quantity. This is particularaly true for complex quantities found in the denominator. The final form of the complex answer should have all phase information in the numerator.]