**CHAPTER V**  
One-Dimensional Potential Wells and Barriers  
Part I. The Delta Function Potential

**Introduction**

So far, we have encountered two very different types of solution to the time independent Schrödinger equation. In the case of the infinite square well the Schrödinger equation

\[
\widehat{H}\psi_n(x) = E_n\psi_n(x)
\]

(5.1)
gives us a set of normalizable, orthogonal wave functions (or eigenfunctions) \(\psi_n(x)\), each associated with a specific, discrete energy value (or eigenvalue) \(E_n\). The most general time-dependent solution to the Schrödinger equation in this case is a linear combination of these eigenfunctions of the form

\[
\Psi(x, t) = \sum_n c_n e^{-i\omega_n t} \psi_n(x)
\]

(5.2)

where \(\omega_n = E_n/\hbar\), and where the expansion coefficients, \(c_n\), are determined from the wave function at time \(t = 0\) from the equation

\[
c_n = \int_{-\infty}^{+\infty} \Psi(x, 0) \psi_n^*(x) \, dx
\]

(5.3)

These coefficients must also satisfy the normalization condition

\[
\sum_n |c_n|^2 = 1
\]

(5.4)

The expectation value of energy of this system is given by

\[
\langle \hat{H} \rangle = \sum_n |c_n|^2 E_n
\]

(5.5)

In the case of the free particle, however, we encountered a solution to Schrödinger's time-independent equation which was associated with a continuous range of possible energy values (for \(E > 0\)). The most general normalizable solution to the Schrödinger equation in this case is the integral

\[
\Psi(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \phi(k) e^{i(kx-\omega t)} \, dk
\]

(5.6)

where

\[
\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \Psi(x, 0) e^{-ikx} \, dx
\]

(5.7)
Note: Although the “eigenfunctions” of the momentum operator are not properly normalizable wavefunctions, we can express the expectation value of the energy for a free particle using the equation

\[ \langle \hat{H} \rangle = \int_{-\infty}^{+\infty} |\phi(k)|^2 \left( \frac{\hbar^2}{2m} \right)^2 dk \quad (5.8) \]

This is analogous to the way we have computed other expectation values, except that, here, we are using the momentum dependent functions for the free particle.

You should perform this operation starting with the free-particle wave functions \( \Psi(x, t) \), and the Hamiltonian operator \( \hat{H} = \frac{\hat{p}^2}{2m} \) expressed in configuration space (where \( \hat{p} = -i\hbar \partial / \partial x \)) and show that you obtain the result above. Notice that this expression is just the expectation value of the energy (expressed in terms of the momentum \( p = \hbar k \)) in momentum space.

**The Delta-Function Potential Well Problem**

We now want to consider a particularly simple (though somewhat unrealistic) problem which exhibits both of these types of solutions - the delta-function potential problem. In this problem, we assume that the potential energy is of the form

\[ V(x) = -\alpha \delta(x) \quad (5.9) \]

which acts something like an infinitely narrow, infinitely deep potential well in an otherwise constant potential background. We will approach the solution to this problem much like we did in the case of the infinite square well.

Schrödinger's time-independent equation for this problem is

\[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + V(x)\psi(x) = E\psi(x) \quad (5.10) \]

where \( V(x) = -\alpha \delta(x) \). This potential function is somewhat unique and looks something like what is shown in the diagram below, with the effective depth of the well going to negative infinity, but with the width of the well being so narrow that the area of the well is unity! This means that for a \( \delta \)-function potential, region II in the diagram below has zero width. We simply draw this potential with a finite width to be able to think clearly about what we are doing. Because region II has zero width, we only need to be concerned about the wave function in regions I and III.
We must now find the solution to Schrödinger's equation in regions I and III. In each of these regions we must consider two different cases, one where \( E > 0 \) and one where \( E < 0 \). We will first examine the case where \( E < 0 \).

**The Delta-Function Potential Well Solution for \( E < 0 \)**

In region I and III, the Schrödinger equation is

\[
-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) = E\psi(x) \tag{5.11}
\]

or

\[
\frac{\partial^2}{\partial x^2} \psi(x) = -\frac{2mE}{\hbar^2} \psi(x) = -k^2\psi(x) \tag{5.12}
\]

which has a solution of the form

\[
\psi(x) = Ae^{\lambda x} \tag{5.13}
\]

Plugging this back into the Schrödinger equation reveals that

\[
\lambda = \pm ik \tag{5.14}
\]

In the case where \( E < 0 \), \( k \) is imaginary, so we write

\[
k = \sqrt{-\frac{2m|E|}{\hbar^2}} = \frac{i\kappa}{\hbar^2} \tag{5.15}
\]

and the solution becomes

\[
\psi_I(x) = Ae^{+\kappa x} + Be^{-\kappa x} \quad \text{for } x < 0 \tag{5.16}
\]

\[
\psi_{III}(x) = Fe^{+\kappa x} + Ge^{-\kappa x} \quad \text{for } x > 0 \tag{5.17}
\]
Now, in region I, where \( x < 0 \), the solution blows up as \( x \to -\infty \), unless \( B = 0 \). Likewise, in region III, where \( x > 0 \), the solution blows up as \( x \to +\infty \), unless \( F = 0 \). This means that the solutions in region I and III are

\[
\psi_I(x) = Ae^{+\kappa x} \quad \text{for} \quad x < 0 \tag{5.18}
\]

\[
\psi_{III}(x) = Ge^{-\kappa x} \quad \text{for} \quad x > 0 \tag{5.19}
\]

As we have discussed earlier, the wave function must be continuous to have meaning as a probability amplitude. This means that for a \( \delta \)-function potential where region II in the diagram above has no real width, the wave function in region I must have the same value as the wave function in region III at the point where \( x = 0 \). This requires that \( A = G \), so the solution to Schrödinger's equation is

\[
\psi_I(x) = Ae^{+\kappa x} \quad \text{for} \quad x < 0 \tag{5.20}
\]

\[
\psi_{III}(x) = Ae^{-\kappa x} \quad \text{for} \quad x > 0 \tag{5.21}
\]

or

\[
\psi_{I,III}(x) = Ae^{-\kappa|x|} \tag{5.22}
\]

The second condition on the wave function solution to Schrödinger's equation is that the first derivative of the wave function must be continuous for piecewise-continuous potentials. However, in the case of the delta-function potential, the potential is \textit{not} piecewise continuous - it is infinite. Let's look again at the requirement imposed on the first derivative of the wave function by the Schrödinger equation. As you will remember, we integrated the Schrödinger equation with respect to \( x \) over a small interval \( \Delta \varepsilon \)

\[
-\frac{\hbar^2}{2m} \int_{x_0-\epsilon}^{x_0+\epsilon} \frac{\partial^2 \psi(x)}{\partial x^2} \, dx + \int_{x_0-\epsilon}^{x_0+\epsilon} V(x) \psi(x) \, dx = E \int_{x_0-\epsilon}^{x_0+\epsilon} \psi(x) \, dx \tag{5.23}
\]

In the limit as \( \Delta \varepsilon \to 0 \) the integral over the wave function must go to zero, since it is a continuous, single-valued function. The integral of the second derivative of the wave function is just the first derivative, so that we are left with

\[
\lim_{\Delta \varepsilon \to 0} \left. \frac{\partial \psi(x)}{\partial x} \right|_{x_0-\epsilon}^{x_0+\epsilon} = \lim_{\Delta \varepsilon \to 0} \frac{2m}{\hbar^2} \int_{x_0-\epsilon}^{x_0+\epsilon} V(x) \psi(x) \, dx \tag{5.24}
\]

As pointed out in the last chapter, the integral of the product of the potential energy function and the wave function can be represented by the diagram shown for piecewise-continuous potential energy functions (i.e., those having a finite number of finite steps).
However, for potential energy functions which have infinite steps, such as the infinite square well and the delta-function potential, the first derivative of the wave function may not be continuous at a boundary. In fact, using the definition of the delta-function, we can evaluate the integral of the product of the wave function and the potential energy function to obtain

$$
\lim_{\Delta \epsilon \to 0} \frac{\partial \psi(x)}{\partial x} \bigg|^{+\epsilon}_{-\epsilon} = \lim_{\Delta \epsilon \to 0} \frac{2m}{\hbar^2} \int_{-\epsilon}^{+\epsilon} -\alpha \delta(x) \psi(x) \, dx = -\frac{2m\alpha}{\hbar^2} \psi(0) \quad (5.25)
$$

which we write as

$$
\lim_{\Delta \epsilon \to 0} \left( \frac{\partial \psi(x)}{\partial x} \bigg|^{+\epsilon}_{-\epsilon} - \frac{\partial \psi(x)}{\partial x} \bigg|^{-\epsilon}_{+\epsilon} \right) = -\frac{2m\alpha}{\hbar^2} \psi(0) = -\frac{2m\alpha}{\hbar^2} A \quad (5.26)
$$

Now the first partial derivative is evaluated in the region $x > 0$, while the second partial is evaluated in the region $x < 0$, giving

$$
\lim_{\Delta \epsilon \to 0} \left( -\kappa A e^{-\kappa x} \bigg|^{+\epsilon}_{+\epsilon} - (+\kappa A e^{+\kappa x}) \bigg|^{-\epsilon}_{-\epsilon} \right) = -\frac{2m\alpha}{\hbar^2} A \quad (5.27)
$$

$$
\lim_{\Delta \epsilon \to 0} \left( -2\kappa A e^{-\kappa \epsilon} \right) = -\frac{2m\alpha}{\hbar^2} A \quad (5.28)
$$

which in the limit as $\epsilon \to 0$ fixes the value of $\kappa$ (and therefore the energy $E$) according to the equation

$$
\kappa = \frac{m\alpha}{\hbar^2} = \sqrt{\frac{2m|E|}{\hbar^2}} \quad (5.29)
$$
where $\alpha$ is the “depth” of the delta-function potential. We see from this equation that there is only one allowed energy for $E < 0$ given by

$$E = -\frac{m^2 \alpha^2}{\hbar^4} \cdot \frac{\hbar^2}{2m} = -\frac{m\alpha^2}{2\hbar^2}$$

(5.30)

The only thing remaining for the case where $E < 0$ is to normalize the wave function and determine $A$

$$\int_{-\infty}^{0} A^2 e^{2\kappa x} dx + \int_{0}^{+\infty} A^2 e^{-2\kappa x} dx = 1$$

(5.31)

$$\left. \frac{A^2 e^{2\kappa x}}{2\kappa} \right|_{0}^{+\infty} + \left. \frac{A^2 e^{-2\kappa x}}{-2\kappa} \right|_{-\infty}^{0} = 1$$

(5.32)

$$\frac{A^2}{2\kappa} \left[ e^{2\kappa x} \right]_{-\infty}^{0} - e^{-2\kappa x} \left. \right|_{0}^{+\infty} = 1$$

(5.33)

$$\frac{A^2}{2\kappa} [(1 - 0) - (0 - 1)] = \frac{A^2}{\kappa} = 1$$

(5.34)

$$\Rightarrow A = \sqrt{\kappa}$$

(5.35)

Thus, for the delta-function potential, we have only one eigenstate corresponding to an energy $E < 0$ given by

$$\psi_E(x) = \sqrt{\kappa} e^{-\kappa|x|} \quad E = -\frac{m\alpha^2}{2\hbar^2}$$

(5.36)

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**Problem 5.1 The Double Delta Function Potential**

Consider the double delta function potential energy function given by

$$V(x) = -\alpha [\delta(x + a) + \delta(x - a)]$$

(a) Sketch this potential energy function.

(b) Show that there are at most two possible bound state energies (i.e., energies where $E < 0$) allowed. Schematically plot the wave function corresponding to each of these two possible solutions. Do both solutions always exist?

(c) In particular, find the allowed energies for the two cases $\alpha = \hbar^2/ma$ and $\alpha = \hbar^2/4ma$, and sketch the corresponding wave functions.
The Delta-Function Potential Well Solution for $E > 0$

In region I and III, the Schrödinger equation is, again

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) = E\psi(x)$$ (5.37)

or

$$\frac{\partial^2}{\partial x^2} \psi(x) = -\frac{2mE}{\hbar^2} \psi(x) = -k^2 \psi(x)$$ (5.38)

which has a solution of the form

$$\psi(x) = Ae^{\lambda x}$$ (5.39)

Plugging this back into the Schrödinger equation reveals that

$$\lambda = \pm ik$$ (5.40)

In the case where $E > 0$, $k$ is real, given by

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$ (5.41)

and the solution becomes

$$\psi_I(x) = Ae^{+ikx} + Be^{-ikx} \quad \text{for } x < 0$$ (5.42)

$$\psi_{III}(x) = Fe^{+ikx} + Ge^{-ikx} \quad \text{for } x > 0$$ (5.43)

The time-dependent form of these two equations have the form of two traveling sinusoidal waves moving in opposite directions. Let's assume that we are dealing with a particle (or group of particles) that are originating in the negative half-plane (i.e., in the region where $x < 0$). The particles will move in from the left, encounter the delta-function potential at $x = 0$, and will either continue moving in the $+x$ direction (i.e., they are transmitted through the region of potential change), or will be reflected and move back in a $-x$ direction. This means that in the region where $x < 0$ we must allow for the possibility of two opposite going waves (or particles), but in the region where $x > 0$ there is only one possibility - the particle (or wave) moves only in the $+x$ direction. This means that we must require that $G = 0$, based upon the initial conditions (the assumption that particles are originating in the negative half-plane). This leaves us with the equations

$$\psi_I(x) = Ae^{+ikx} + Be^{-ikx} \quad \text{for } x < 0$$ (5.44)

$$\psi_{III}(x) = Fe^{+ikx} \quad \text{for } x > 0$$ (5.45)
Since the wave function must be continuous, these two equations must be equal at $x = 0$, giving the condition

$$A + B = F \quad (5.46)$$

The second condition on the wave function solution to Schrödinger’s equation is that the first derivative of the wave function must also be continuous for piecewise-continuous potentials. But again, the potential is not piecewise continuous - it is infinite. As before, we integrate the Schrödinger equation with respect to $x$ over a small interval $\Delta \epsilon$.

\[
-\frac{\hbar^2}{2m} \int_{x_0-\epsilon}^{x_0+\epsilon} \frac{\partial^2 \psi(x)}{\partial x^2} \, dx + \int_{x_0-\epsilon}^{x_0+\epsilon} V(x) \psi(x) \, dx = E \int_{x_0-\epsilon}^{x_0+\epsilon} \psi(x) \, dx \quad (5.47)
\]

Again, in the limit as $\Delta \epsilon \to 0$ the integral over the wave function must go to zero, the integral of the second derivative of the wave function is just the first derivative, and so we are left with

$$\lim_{\Delta \epsilon \to 0} \frac{\partial \psi(x)}{\partial x} \bigg|_{x_0-\epsilon}^{x_0+\epsilon} = \lim_{\Delta \epsilon \to 0} \frac{2m}{\hbar^2} \int_{x_0-\epsilon}^{x_0+\epsilon} V(x) \psi(x) \, dx \quad (5.48)$$

Using the definition of the delta-function, we can evaluate the integral of the product of the wave function and the potential energy function to obtain

$$\lim_{\Delta \epsilon \to 0} \frac{\partial \psi(x)}{\partial x} \bigg|_{x_0-\epsilon}^{x_0+\epsilon} = \lim_{\Delta \epsilon \to 0} \frac{2m}{\hbar^2} \int_{-\epsilon}^{+\epsilon} -\alpha \delta(x) \psi(x) \, dx = -\frac{2m \alpha}{\hbar^2} \psi(0) \quad (5.49)$$

which we write as

$$\lim_{\Delta \epsilon \to 0} \left( \frac{\partial \psi(x)}{\partial x} \bigg|_{x_0+\epsilon} - \frac{\partial \psi(x)}{\partial x} \bigg|_{x_0-\epsilon} \right) = -\frac{2m \alpha}{\hbar^2} (A + B) \quad (5.50)$$

Now the first partial derivative is evaluated in the region $x > 0$, while the second partial is evaluated in the region $x < 0$, giving

$$\lim_{\Delta \epsilon \to 0} \left( +ikFe^{+ik} \right|_{x_0+\epsilon} - \left( +ikAe^{+ik} + (-ik)Be^{-ik} \right|_{x_0-\epsilon} \right) = -\frac{2m \alpha}{\hbar^2} (A + B)$$

$$\lim_{\Delta \epsilon \to 0} \left( +ikFe^{+ik} - \left( +ikAe^{-ik} - ikBe^{+ik} \right|_{x_0-\epsilon} \right) = -\frac{2m \alpha}{\hbar^2} (A + B)$$

$$\lim_{\Delta \epsilon \to 0} ike^ {ik} - Ae^{-ik} = -\frac{2m \alpha}{\hbar^2} (A + B) \quad (5.51)$$

Now, in the limit this reduces to

$$F + B - A = \frac{i2m \alpha}{\hbar^2 k}(A + B) = 2i \beta (A + B) \quad (5.52)$$
where $\beta = m\alpha/\hbar^2k$. But from the condition on the continuity of the wave function we know that $F = A + B$, so this last equation reduces to

$$B = i\beta(A + B)$$

(5.53)

or

$$B = \frac{i\beta}{1 - i\beta}A$$

(5.54)

Solving for $F$ gives, since $F = A + B$,

$$F = \frac{1}{1 - i\beta}A$$

(5.55)

It should be obvious that the amplitude $A$ is related to the probability of measuring an incoming particle, while the amplitude $B$ is related to the probability of measuring a reflected particle and the amplitude $F$ to measuring a transmitted particle. To determine a precise relationship between these quantities we return to the concept of the probability current density defined by the equation

$$j(x, t) = -\frac{i\hbar}{2m} \left[ \Psi^*(x, t) \frac{\partial \Psi(x, t)}{\partial x} - \Psi(x, t) \frac{\partial \Psi^*(x, t)}{\partial x} \right]$$

(5.56)

As you will recall, this equation has the form

$$\tilde{j}(\vec{r}, t) = -\frac{i\hbar}{2m} \left[ \Psi^*(\vec{r}, t) \vec{\nabla} \Psi(\vec{r}, t) - \Psi(\vec{r}, t) \vec{\nabla} \Psi^*(\vec{r}, t) \right]$$

(5.57)

in three-dimensions, which satisfies the continuity equation

$$\frac{\partial \rho(\vec{r})}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

(5.58)

This equation states that the change in the probability density at some point is associated with the probability current flowing away from that point. If we integrate the continuity equation over some enclosed volume $V$, we obtain

$$-\int_V \frac{\partial \rho}{\partial t} \, dv = \int_V \vec{\nabla} \cdot \vec{j} \, dv \approx \oint_S \vec{j} \cdot \hat{n} \, ds$$

(5.59)

where the last integral is due to Stokes' theorem. Thus, if the probability of finding a particle within an enclosed volume decreases in time, this corresponds to a probability flux, the sum of all probability currents flowing out of an enclosed volume. Thus in three-dimensions $\vec{j}$ corresponds to the relative number of particles per unit volume flowing outward through the enclosing surface $S$, times the speed at which the particles leave. This has units of the number of particles passing through a given cross-sectional area per unit time. Thus, if we consider a stream of particles leaving an enclosed volume and incident upon some surface area $dS$, we would designate $j_{\text{incident}}$ as the relative number of particles per unit volume that were approaching the surface, $j_{\text{reflected}}$ as the
relative number of particles per unit volume that were reflected from that surface and \( j_{\text{transmitted}} \) as the relative number of particles per unit volume that were passing through the surface. We define the transmission and reflection coefficients in terms of these current densities according to the equations

\[
R \equiv \left| \frac{j_{\text{reflected}}}{j_{\text{incident}}} \right| \quad \text{and} \quad T \equiv \left| \frac{j_{\text{transmitted}}}{j_{\text{incident}}} \right|
\] (5.60)

The solutions we have obtained for our delta function potential are traveling plane waves of the form \( \Psi(x, t) = A e^{i(kx - \omega t)} \). The probability current density for a plane wave is given by

\[
\vec{j} = -\frac{i\hbar}{2m} \left[ \Psi^*(x, t) \frac{\partial}{\partial x} \Psi(x, t) - \Psi(x, t) \frac{\partial}{\partial x} \Psi^*(x, t) \right]
\] (5.61)

or

\[
\vec{j} = -\frac{i\hbar}{2m} \left[ A^* e^{-i(kx - \omega t)} \frac{\partial}{\partial x} Ae^{i(kx - \omega t)} - Ae^{i(kx - \omega t)} \frac{\partial}{\partial x} A^* e^{-i(kx - \omega t)} \right]
\] (5.62)

\[
\vec{j} = -\frac{i\hbar}{2m} [A^* e^{-i(kx - \omega t)} ikAe^{i(kx - \omega t)} - Ae^{i(kx - \omega t)} (-ik)A^* e^{-i(kx - \omega t)}]
\] (5.63)

\[
\vec{j} = -\frac{i\hbar}{2m} [ik|A|^2 - (-ik)|A|^2]
\] (5.64)

\[
\vec{j} = -\frac{i\hbar}{2m} [2ik|A|^2]
\] (5.65)

\[
\vec{j} = \frac{\hbar k}{m} |A|^2
\] (5.66)

This has the form

\[
\vec{j} = \frac{p}{m} |A|^2 = v|A|^2
\] (5.67)

and will be positive or negative depending upon the sign of \( k \).

This means that the incident, reflected and transmitted probability current densities for plane waves are given, respectively, by

\[
\vec{j}_{\text{inc}} = +\frac{\hbar k}{m} |A|^2
\] (5.68)

\[
\vec{j}_{\text{refl}} = -\frac{\hbar k}{m} |B|^2
\] (5.69)
\[ \tilde{J}_{\text{trans}} = \frac{\hbar k}{m} |F|^2 \]  

(5.70)

Thus, the reflection coefficient for our delta function potential is given by

\[ R = \frac{|B|^2}{|A|^2} = \frac{\beta^2}{1 + \beta^2} = \frac{1}{1 + 1/\beta^2} \]  

(5.71)

and the transmission coefficient is given by

\[ T = \frac{|F|^2}{|A|^2} = \frac{1}{1 + \beta^2} \]  

(5.72)

Notice that in this particular problem, the value of \( k = \sqrt{2m(E - V)/\hbar^2} \) is the same in regions I and III of figure 4.1, but in other problems, the value of \( k \) may be different. In those cases, the transmission and reflection coefficients depend upon the value of \( k \) in the respective regions.

You should notice that the sum of the reflection and transmission probabilities equal unity as it must. Now using the definition of \( \beta \) and \( k \), we find that

\[ \beta^2 = \frac{m^2 \alpha^2}{\hbar^4 k^2} = \frac{m^2 \alpha^2}{\hbar^4 (2mE/\hbar^2)} = \frac{m \alpha^2}{2\hbar^2 E} \]  

(5.73)

So we can express the reflection and transmission coefficients for the delta-function potential well in terms of the energy and mass of the incident particle:

\[ R = \frac{1}{1 + (2\hbar^2 E/m \alpha^2)} \]  

(5.74)

\[ T = \frac{1}{1 + (m \alpha^2/2\hbar^2 E)} \]  

(5.75)

Thus, for situations where the energy of the incident particle is small, there is a relatively high probability for reflection at \( x = 0 \), but as the energy of the incident particle increases, the probability for reflection goes down.

However, there is one small problem we have not yet addressed. Plane wave solutions for the free particle are not normalizable, so are not really acceptable solutions! At first glance, this might seem a bit problematic, but we have shown that an acceptable solution can be formed from a combination of plane wave solutions using a Fourier integral. This means that each component of the Fourier integral will be transmitted or reflected with different probability, because each component has a slightly different energy. We can, however, obtain an estimate of the approximate transmission or reflection coefficient based upon the more prevalent component of the wave packet.
The Delta-Function Potential Barrier Problem

For the case of a delta-function potential barrier, only the sign of the potential energy function is changed, so that the potential energy function has the form

\[ V(x) = +\alpha \delta(x) \]  

which acts something like an infinitely narrow, infinitely tall potential barrier in an otherwise constant potential background. It should be obvious that there in no bound state solution for this case (i.e., where \( E < 0 \)). We only have to consider the case where \( E > 0 \). For this case all we have to do in order to find the solution for the infinite barrier is to change the sign of \( \alpha \). But the reflection and transmission coefficients are a function only of the square of \( \alpha \), so that we obtain the same result as for the potential well! Thus, whether we are dealing with a delta-function well or barrier, the transmission and scattering coefficients are identical. This means that a quantum mechanical particle can penetrate a potential barrier of infinite height - provided it is not too wide! This phenomena is called tunneling. We will see how this same situation plays out for finite width barriers and wells in the next chapter.