Equations of State

1. Derive an expression for the thermal coefficient of expansion $\beta$, and for the isothermal coefficient of compressibility $\kappa$ for a gas which obeys
   a) the ideal gas equation of state, and
   b) the van der Waal equation of state.

2. For many solids and liquids, $\beta$ and $\kappa$ can be treated as constants (over a limited range of temperature values). Show that the equation of state for such a substance is given by
   \[ V = C e^{\beta T - \kappa P} \]

   Show that this equation can be approximated by the equation
   \[ V = V_o [1 + \beta(T - T_o) - \kappa(P - P_o)] \]
   for values of $T$ and $P$ close to the values where $V_o$ is the volume where $T = T_o$ and $P = P_o$.

3. A sample of liquid oxygen is heated from 60K to 90K (the boiling point of O$_2$) in a container with a fixed volume. What is the final pressure of the oxygen if its initial pressure was $10^5$ Pa (or approximately one atmosphere)? Assume that $\beta = 3.96 \times 10^{-3}/K$ and that $k = 1.42 \times 10^{-9}/Pa$.

4. The pressure differential in a static fluid is given by
   \[ dp = -\rho g \, dz \]

   where $\rho$ is the density, $g$ is the acceleration of gravity, and $z$ is positive upward. Assume that the atmosphere is such a static fluid and that you can approximate the atmosphere as an ideal gas. Show that for an isothermal atmosphere, the pressure varies with altitude according to the relation
   \[ P = P_o e^{-(z-z_o)/\xi} \]

   where $\xi$ is called the “scale height” and is given by $R_u T_o / \mu g$ with $R_u$ being the universal gas constant, $T_o$ the temperature at $z = z_o$, $g$ the gravitational constant, and $\mu$ the molar mass.